TermMax - Range Order

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Abstract

TermMax introduces a range order lending model inspired by Uniswap V3's concentrated liquidity. Each USDC can be split into two tokens—FT (principal) and XT (interest). By defining multiple cut points, users create a piecewise exchange rate between FT and XT, aligning different interest levels within specified ranges.

1 Notations

Symbol	Description
$\mathcal{R}_{\mathbf{ft}}$	FT reserve in the range order
$\mathcal{R}_{\mathbf{xt}}$	XT reserve in the range order
$\mathcal{L}$	liquidity of the range order
$\mathcal{P}$	current FT to XT price of the range order
$\mathcal{P}_0$	left boundary of the FT to XT price range in single-range order
$\mathcal{P}_1$	right boundary of the FT to XT price range in single-range order
$\mathcal{P}_i$	the i^{th} price in multiple-range order
r_0	left boundary of the APR range in single-range order
r_1	right boundary of the APR range in single-range order
r_i	the i^{th} APR in multiple-range order
d_m	days to maturity
θ	time ratio according to days to maturity
$\mathcal{F}_{maker}$	maker fee ratio
$\mathcal{F}_{taker}$	taker fee ratio

Table 1: Symbol Table Notation

$$\theta := \frac{d_m}{365} \tag{1}$$

$$\mathcal{P} := r\theta \tag{2}$$

2 Curve of Range Order

2.1 Preliminary

Eq. 3 shows the curve of concentrated liquidity on Uniswap V3 with XT (X token) and FT (Fixed-Rate Token) in the pool. By replace the price $\mathcal{P}$ according to Eq. 2, we can derive Eq. 4 as the base curve of a single segment for range orders on TermMax.

$$(\mathcal{R}_{\mathbf{xt}} + \frac{\mathcal{L}}{\sqrt{\mathcal{P}_1}})(\mathcal{R}_{\mathbf{ft}} + \mathcal{L}\sqrt{\mathcal{P}_0}) = \mathcal{L}^2 \tag{3}$$

$$(\mathcal{R}_{\mathbf{xt}} + \frac{\mathcal{L}}{\sqrt{r_1\theta}})(\mathcal{R}_{\mathbf{ft}} + \mathcal{L}\sqrt{r_0\theta}) = \mathcal{L}^2 \tag{4}$$

With the curve defined as Eq. 4, we can derive the relationship between current price ($\mathcal{P}$) and token reserves ($\mathcal{R}_{\mathbf{xt}}$, $\mathcal{R}_{\mathbf{ft}}$).

Theorem 2.1. *Given current price ($\mathcal{P}$), APR (r) and time ratio (θ).*

$$(\mathcal{R}_{\mathbf{xt}} + \frac{\mathcal{L}}{\sqrt{r_1\theta}})(\mathcal{R}_{\mathbf{ft}} + \mathcal{L}\sqrt{r_0\theta}) = \mathcal{L}^2$$

iff

$$\mathcal{P} = r\theta = -\frac{d\mathcal{R}_{\mathbf{ft}}}{d\mathcal{R}_{\mathbf{xt}}} = \frac{(\mathcal{R}_{\mathbf{ft}} + \mathcal{L}\sqrt{r_0\theta})}{(\mathcal{R}_{\mathbf{xt}} + \frac{\mathcal{L}}{\sqrt{r_1\theta}})} \tag{5}$$

Proof. Let

$$\begin{cases} \mathcal{V}_{\mathcal{R}_{\mathbf{xt}}} := \mathcal{R}_{\mathbf{xt}} + \frac{\mathcal{L}}{\sqrt{r_1\theta}} \\ \mathcal{V}_{\mathcal{R}_{\mathbf{ft}}} := \mathcal{R}_{\mathbf{ft}} + \mathcal{L}\sqrt{r_0\theta} \end{cases}$$

Then

$$\begin{aligned} &\frac{d\mathcal{R}_{\mathbf{ft}}}{d\mathcal{R}_{\mathbf{xt}}} \\ =&\frac{d(\mathcal{R}_{\mathbf{ft}} + \mathcal{L}\sqrt{r_0\theta})}{d(\mathcal{R}_{\mathbf{xt}} + \frac{\mathcal{L}}{\sqrt{r_1\theta}})} \\ =&\frac{d\mathcal{V}_{\mathcal{R}_{\mathbf{ft}}}}{d\mathcal{V}_{\mathcal{R}_{\mathbf{xt}}}} \\ =&\frac{d}{d\mathcal{V}_{\mathcal{R}_{\mathbf{xt}}}}\left(\frac{\mathcal{L}^2}{\mathcal{V}_{\mathcal{R}_{\mathbf{xt}}}}\right) \\ =&\left(\frac{-\mathcal{L}^2}{\mathcal{V}^2_{\mathcal{R}_{\mathbf{xt}}}}\right) \\ =&\left(\frac{-\mathcal{V}_{\mathcal{R}_{\mathbf{xt}}}\mathcal{V}_{\mathcal{R}_{\mathbf{ft}}}}{\mathcal{V}^2_{\mathcal{R}_{\mathbf{xt}}}}\right) \\ =&\left(\frac{-\mathcal{V}_{\mathcal{R}_{\mathbf{ft}}}}{\mathcal{V}_{\mathcal{R}_{\mathbf{xt}}}}\right) \\ =&\frac{-(\mathcal{R}_{\mathbf{ft}} + \mathcal{L}\sqrt{r_0\theta})}{(\mathcal{R}_{\mathbf{xt}} + \frac{\mathcal{L}}{\sqrt{r_1\theta}})} \end{aligned}$$

□

2.2 Single-Range Order

When Alice places a single-range order, she will determine two cut points ($\mathcal{R}_{\mathbf{xt}0} = 0$, r_1) and ($\mathcal{R}_{\mathbf{xt}1}$, r_0) which specify the expected APR to be matched according to $\mathcal{R}_{\mathbf{xt}}$ in the order. With the given cut points, liquidity ($\mathcal{L}$) can be derived according to Eq. 4.

By substituting the cut points into Eq. 4 respectively, we can derive $\mathcal{L}$ with Eq. 6.

$$(\mathcal{R}_{\mathbf{xt}1} + \frac{\mathcal{L}}{\sqrt{r_1\theta}})(\mathcal{R}_{\mathbf{ft}1} + \mathcal{L}\sqrt{r_0\theta}) = \mathcal{L}^2 \tag{6}$$

where

$$\mathcal{R}_{\mathbf{ft}1} = -r_0\theta(\mathcal{R}_{\mathbf{xt}1} + \frac{\mathcal{L}}{\sqrt{r_1\theta}}) - \mathcal{L}\sqrt{r_0\theta}$$

Therefore, we can define a single-range curve with $\mathcal{L}^2$, $\mathcal{R}_{\mathbf{xt}}$ (equals to $\mathcal{R}_{\mathbf{xt}0}$), and $\beta = \frac{\mathcal{L}}{\sqrt{r_1\theta}}$, which will be stored on-chain for pricing the range order.

2.3 Adapting Fixed Income Model

Owing to the nature of fixed income model, the price of FT will increase and the price of XT will decrease day by day under the same APR when the time approaches maturity date gradually. Therefore, the exchange rate of XT and FT will change day by day as well. Since the pricing curve will changed periodically, the pricing curve will be calculated whenever the transaction executed. The new pricing curve will be calculated based on the stored $\mathcal{L}^2$, $\mathcal{R}_{\mathbf{xt}}$, and $\beta = \mathcal{L}\sqrt{r_0\theta}$ and current time ratio (θ) as shown in Theorem 2.2.

The $\mathcal{R}_{\mathbf{xt}}$ stands for the actual XT reserve in the range order, therefore, the new XT reserve (${\mathcal{R}_{\mathbf{xt}}}'$) will remain the same. In addition, the FT to XT price should increase according to the time ratio (θ). As a result, the new FT reserve (${\mathcal{R}_{\mathbf{ft}}}'$) should be increased according θ as well according to Theorem 2.1 ($\mathcal{R}_{\mathbf{ft}}$ can be derived according to Eq. 4). Finally, assuming the new $\mathcal{L}'$ equals to $\mathcal{L}\sqrt{\frac{\theta'}{\theta}}$, both ($\mathcal{R}_{\mathbf{xt}}$,$\mathcal{R}_{\mathbf{ft}}$,$\mathcal{L}$) and (${\mathcal{R}_{\mathbf{xt}}}'$,${\mathcal{R}_{\mathbf{ft}}}'$,$\mathcal{L}'$) can satisfy Eq. 4 as shown in Theorem 2.2.

Theorem 2.2.

$$\begin{cases} {\mathcal{R}_{\mathbf{xt}}}' = \mathcal{R}_{\mathbf{xt}} \\ {\mathcal{R}_{\mathbf{ft}}}' = \mathcal{R}_{\mathbf{ft}}\left(\frac{\theta}{\theta'}\right) \\ \mathcal{L}' = \mathcal{L}\sqrt{\frac{\theta'}{\theta}} \end{cases}$$

iff

$$\left(\mathcal{R}_{\mathbf{ft}} + \mathcal{L}\sqrt{r_0\theta}\right)\left(\mathcal{R}_{\mathbf{xt}} + \frac{\mathcal{L}}{\sqrt{r_1\theta}}\right) = \mathcal{L}^2 \leftrightarrow \left({\mathcal{R}_{\mathbf{ft}}}' + \mathcal{L}'\sqrt{r_0\theta'}\right)\left({\mathcal{R}_{\mathbf{xt}}}' + \frac{\mathcal{L}'}{\sqrt{r_1\theta'}}\right) = {\mathcal{L}'}^2$$

Proof.

$$\begin{aligned}
&\left(\mathcal{R}_{\mathbf{ft}} + \mathcal{L}\sqrt{r_0\theta}\right)\left(\mathcal{R}_{\mathbf{xt}} + \frac{\mathcal{L}}{\sqrt{r_1\theta}}\right) = \mathcal{L}^2 \\
\Leftrightarrow &\left({\mathcal{R}_{\mathbf{ft}}}'\left(\frac{\theta}{\theta'}\right) + \mathcal{L}\sqrt{r_0\theta}\right)\left({\mathcal{R}_{\mathbf{xt}}}' + \frac{\mathcal{L}}{\sqrt{r_1\theta}}\right) = \mathcal{L}^2 \\
\Leftrightarrow &\left({\mathcal{R}_{\mathbf{ft}}}' + \left(\frac{\theta'}{\theta}\right)\mathcal{L}\sqrt{r_0\theta}\right)\left({\mathcal{R}_{\mathbf{xt}}}' + \left(\frac{\mathcal{L}}{\sqrt{r_1\theta}}\right)\right) = \left(\frac{\theta'}{\theta}\right)\mathcal{L}^2 \\
\Leftrightarrow &\left({\mathcal{R}_{\mathbf{ft}}}' + \left(\mathcal{L}\sqrt{\frac{\theta'}{\theta}}\right)\sqrt{r_0\theta'}\right)\left({\mathcal{R}_{\mathbf{xt}}}' + \left(\frac{\mathcal{L}}{\sqrt{r_1\theta}}\right)\right) = \left(\frac{\theta'}{\theta}\right)\mathcal{L}^2 \\
\Leftrightarrow &\left({\mathcal{R}_{\mathbf{ft}}}' + \left(\mathcal{L}\sqrt{\frac{\theta'}{\theta}}\right)\sqrt{r_0\theta'}\right)\left({\mathcal{R}_{\mathbf{xt}}}' + \left(\frac{\mathcal{L}}{\sqrt{r_1\theta}}\right)\right) = \left(\mathcal{L}\sqrt{\frac{\theta'}{\theta}}\right)^2 \\
\Leftrightarrow &\left({\mathcal{R}_{\mathbf{ft}}}' + \left(\mathcal{L}\sqrt{\frac{\theta'}{\theta}}\right)\sqrt{r_0\theta'}\right)\left({\mathcal{R}_{\mathbf{xt}}}' + \left(\frac{\left(\mathcal{L}\sqrt{\frac{\theta'}{\theta}}\right)\left(\sqrt{\frac{\theta}{\theta'}}\right)}{\sqrt{r_1\theta}}\right)\right) = \left(\mathcal{L}\sqrt{\frac{\theta'}{\theta}}\right)^2 \\
\Leftrightarrow &\left({\mathcal{R}_{\mathbf{ft}}}' + \left(\mathcal{L}\sqrt{\frac{\theta'}{\theta}}\right)\sqrt{r_0\theta'}\right)\left({\mathcal{R}_{\mathbf{xt}}}' + \left(\frac{\mathcal{L}\sqrt{\frac{\theta'}{\theta}}}{\sqrt{r_1\theta'}}\right)\right) = \left(\mathcal{L}\sqrt{\frac{\theta'}{\theta}}\right)^2 \\
\Leftrightarrow &\left({\mathcal{R}_{\mathbf{ft}}}' + \mathcal{L}'\sqrt{r_0\theta'}\right)\left({\mathcal{R}_{\mathbf{xt}}}' + \frac{\mathcal{L}'}{\sqrt{r_1\theta'}}\right) = {\mathcal{L}'}^2
\end{aligned}$$

□

2.4 Extending to Multiple Ranges

In Uniswap V3, users typically set a single price range for their liquidity. TermMax extends this concept by allowing multiple price ranges in as a multiple-range order, offering more granular control over the matched APR for lending or borrowing. For example, if Alice wants to lend 1000 USDC, she might allocate 80% of her funds to a 10%–15% APR range and the remaining 20% to a 15%–40% APR range. Concretely, this involves three cut points—(0, 10%), (800, 15%), and (1000, 40%)—producing two ranges: 10%–15% and 15%–40%. When Bob borrows from Alice's range order, the borrowing APR is determined by how his loan amount intersects with these ranges.

Each range in a TermMax multiple-range order behaves like a concentrated liquidity position in Uniswap V3. The innovation is that multiple such ranges can be combined into one multiple-range order, offering flexible and efficient interest rate control. After users specify the cut points and ranges for their range orders, the liquidity ($\mathcal{L}$) and reserves of XT ($\mathcal{R}_{\mathbf{xt}}$) and β can be derived accordingly for each range to form a pricing curve.

2.4.1 Multiple-Range Order Definition

A multiple-range order can be defined with n cut points: ($\mathcal{R}_{\mathbf{xt}0} = 0$, r_{n-1}), ($\mathcal{R}_{\mathbf{xt}1}$, r_{n-2}), ($\mathcal{R}_{\mathbf{xt}2}$, r_{n-3}), ... ($\mathcal{R}_{\mathbf{xt}n-1}$, r_0),with $n-1$ ranges: $[r_{n-1}$,$r_{n-2}]$, $[r_{n-2}$, $r_{n-3}]$, ... , $[r_1$, $r_0]$.

For range i, TermMax contract stores

$$\mathcal{R}_{\mathbf{xt}i}, \mathcal{L}_i^2, \beta_i = \frac{\mathcal{L}}{\sqrt{r_{n-i-1}\theta}} - \mathcal{R}_{\mathbf{xt}i}, \text{ for } i \in \mathbb{Z}_{n-2} \text{ and } \mathcal{R}_{\mathbf{xt}n-1}$$

To guarantee the prices of range i and range $i+1$ are aligned at $x = \mathcal{R}_{\mathbf{xt}i}$, the relationship between β_i and β_{i+1} needs to be checked as show in Theorem 2.3.

Theorem 2.3. *If the curves of two ranges intersect at $x = \mathcal{R}_{\mathbf{xt}i}$ and the prices are aligned,*

$$\beta_{i+1} = \frac{\mathcal{L}_{i+1}}{\mathcal{L}_i}\left(\mathcal{R}_{\mathbf{xt}i} + \beta_i\right) - \mathcal{R}_{\mathbf{xt}i+1}$$

Proof. First, observe that the $r\theta$ is given by

$$r\theta = -\frac{d\mathcal{R}_{\mathbf{ft}}}{d\mathcal{R}_{\mathbf{xt}}} = \frac{(\mathcal{R}_{\mathbf{ft}} + \mathcal{L}\sqrt{r_{n-i-2}\theta})}{(\mathcal{R}_{\mathbf{xt}} + \frac{\mathcal{L}}{\sqrt{r_{n-i-1}\theta}} - \mathcal{R}_{\mathbf{xt}i})} = \frac{\mathcal{L}^2}{(\mathcal{R}_{\mathbf{xt}} + \frac{\mathcal{L}}{\sqrt{r_{n-i-1}\theta}} - \mathcal{R}_{\mathbf{xt}i})^2} = \frac{\mathcal{L}^2}{(\mathcal{R}_{\mathbf{xt}} + \beta)^2}$$

From the equality of $r\theta$ terms in both ranges, we have

$$\frac{\mathcal{L}_i^2}{(\mathcal{R}_{\mathbf{xt}i+1} + \beta_i)^2} = r_{n-i}\theta = \frac{\mathcal{L}_{i+1}^2}{(\mathcal{R}_{\mathbf{xt}i+1} + \beta_{i+1})^2}$$
$$\Leftrightarrow \beta_{i+1} = \frac{\mathcal{L}_{i+1}}{\mathcal{L}_i}\left(\mathcal{R}_{\mathbf{xt}i+1} + \beta_i\right) - \mathcal{R}_{\mathbf{xt}i+1}$$

□

3 Fee Mechanism

3.1 Overview

When users interact with TermMax, we will charge transaction fees from both range-order makers and takers. For example, if Alice is a range-order maker who places a borrowing range order with constant borrowing rate 20%, and Bob is a lender who wants to take Alice's order. Assuming TermMax will charge 4% of interest as transaction fees from maker and 6% of interest as transaction fees from taker, eventually, Alice will borrow at 20% + 20% * 4% = 20.8% and Bob will lend at 20% - 20% * 6% = 18.8%.

Therefore, to optimize the fee calculation, the original curve of constant 20% will be changed to 18.8% for swap calculation. Then the entire 10% of interest as transaction fees will be charged at 20% after the swap operation. The output result of Alice (maker) and Bob (taker) will be exactly the same as the original curve.

To obtain this optimized calculation, Theorem 2.2 needs to be modified according to the fee mechanism as shown in Eq. 7.

$$\begin{cases} f_{taker} = \mathcal{F}_{taker} \text{ if the taker is a lender} \\ f_{taker} = -\mathcal{F}_{taker} \text{ if the taker is a borrower} \end{cases}$$

$$\begin{cases} \mathcal{R}_{\mathbf{xt}}{}' = \mathcal{R}_{\mathbf{xt}} \\ \mathcal{R}_{\mathbf{ft}}{}' = \frac{\mathcal{R}_{\mathbf{ft}}\left(\frac{\theta}{\theta'}\right)}{1-f_{taker}} \\ \mathcal{L}' = \mathcal{L}\sqrt{\frac{\theta'}{\theta}(1-f_{taker})} \end{cases} \tag{7}$$

4 Swap Between FT and XT

4.1 Overview

Whenever a swap between FT and XT is executed in the range order, the current liquidity ($\mathcal{L}'^2$) and virtual reserves ($\mathcal{V}_{\mathcal{R}_{\mathbf{xt}}}$ and $\mathcal{V}_{\mathcal{R}_{\mathbf{ft}}}$) will be calculated according to Eq. 8 for further swap calculation.

Given $\mathcal{L}^2$ is the stored liquidity value of the range, θ' is the current time ratio, and $\mathcal{R}_{\mathbf{xt}}$ is the current XT reserve in the range order. To simply the calculation, we always define θ as $365/365 = 1$ whenever the order is placed.

$$\begin{cases} \mathcal{L}'^2 = \mathcal{L}^2\theta'(1-f_{taker}) \\ \mathcal{V}_{\mathcal{R}_{\mathbf{xt}}} = \mathcal{R}_{\mathbf{xt}} + \beta \\ \mathcal{V}_{\mathcal{R}_{\mathbf{ft}}} = \frac{\mathcal{L}'^2}{\mathcal{V}_{\mathcal{R}_{\mathbf{xt}}}} \end{cases} \tag{8}$$

4.2 Swap Operations

Before buys FT from a range order, Alice needs to convert the asset into FT and XT first then swap the minted XT to additional FT. In contrast, if Alice want to buy XT with asset, she needs to convert the asset into FT and XT first then swap FT for additional XT.

On the other hand, if Alice want to sell FT for asset, she needs to split the FT to be sold into the principal part and interest part. The interest part of FT will be sold to the range order for exchanging XT. Finally, Alice can combined the principal part of FT and the exchanged XT into asset. The same operations will be executed if Alice wants to sell XT for asset.

4.2.1 Buy $\Delta_{\mathbf{ft}}$ with $\Delta_{\mathbf{xt}}$

$$\Delta_{\mathbf{ft}} = \frac{\mathcal{L}'^2}{\mathcal{V}_{\mathcal{R}_{\mathbf{xt}}} + \Delta_{\mathbf{xt}}} - \mathcal{V}_{\mathcal{R}_{\mathbf{ft}}} \tag{9}$$

4.2.2 Buy $\Delta_{\mathbf{xt}}$ with $\Delta_{\mathbf{ft}}$

$$\Delta_{\mathbf{xt}} = \frac{\mathcal{L}'^2}{\mathcal{V}_{\mathcal{R}_{\mathbf{ft}}} + \Delta_{\mathbf{ft}}} - \mathcal{V}_{\mathcal{R}_{\mathbf{xt}}} \tag{10}$$

4.2.3 Sell FT for Asset

Given FT amount to sell is $\mathcal{A}_{\mathbf{ft}}$,
if $\sigma_{\mathbf{ft}}$ FTs, $\sigma_{\mathbf{xt}}$ XTs are the accumulated amount that have been matched from the previous ranges, where

$$\sigma_{\mathbf{ft}} \geq 0, \sigma_{\mathbf{xt}} \leq 0$$

The final amount of FT needs to be equal to the final amount of XT for redeeming the asset.

$$\begin{aligned} &\mathcal{A}_{\mathbf{ft}} - (\Delta_{\mathbf{ft}} + \sigma_{\mathbf{ft}}) = -(\Delta_{\mathbf{xt}} + \sigma_{\mathbf{xt}}) \\ \Rightarrow &\Delta_{\mathbf{xt}} = \Delta_{\mathbf{ft}} + \sigma_{\mathbf{ft}} - \sigma_{\mathbf{xt}} - \mathcal{A}_{\mathbf{ft}} \\ \Rightarrow &\frac{\mathcal{L}'^2}{\mathcal{V}_{\mathcal{R}_{\mathbf{ft}}} + \Delta_{\mathbf{ft}}} - \mathcal{V}_{\mathcal{R}_{\mathbf{xt}}} = \Delta_{\mathbf{ft}} + \sigma_{\mathbf{ft}} - \sigma_{\mathbf{xt}} - \mathcal{A}_{\mathbf{ft}} \end{aligned}$$

Then

$$\Delta_{\mathbf{ft}} = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

where

$$\begin{cases} a & = 1 \\ b & = (\sigma_{\mathbf{ft}} - \sigma_{\mathbf{xt}} - \mathcal{A}_{\mathbf{ft}}) + \mathcal{V}_{\mathcal{R}_{\mathbf{ft}}} + \mathcal{V}_{\mathcal{R}_{\mathbf{xt}}} \\ c & = \mathcal{V}_{\mathcal{R}_{\mathbf{ft}}} (\sigma_{\mathbf{ft}} - \sigma_{\mathbf{xt}} - \mathcal{A}_{\mathbf{ft}}) \end{cases}$$

4.2.4 Sell XT for Asset

Given ft amount to sell is $\mathcal{A}_{\mathbf{xt}}$ If $\sigma_{\mathbf{xt}}$ XTs, $\sigma_{\mathbf{ft}}$ FTs have already been matched, where

$$\sigma_{\mathbf{xt}} \geq 0, \sigma_{\mathbf{ft}} \leq 0$$

The final amount of FT needs to be equal to the final amount of XT for redeeming the asset.

$$\begin{aligned} & \mathcal{A}_{\mathbf{xt}} - (\Delta_{\mathbf{xt}} + \sigma_{\mathbf{xt}}) = -(\Delta_{\mathbf{ft}} + \sigma_{\mathbf{ft}}) \\ \Rightarrow & \Delta_{\mathbf{ft}} = \Delta_{\mathbf{xt}} + \sigma_{\mathbf{xt}} - \sigma_{\mathbf{ft}} - \mathcal{A}_{\mathbf{xt}} \\ \Rightarrow & \frac{\mathcal{L}'^2}{\mathcal{V}_{\mathcal{R}_{\mathbf{xt}}} + \Delta_{\mathbf{xt}}} - \mathcal{V}_{\mathcal{R}_{\mathbf{ft}}} = \Delta_{\mathbf{xt}} + \sigma_{\mathbf{xt}} - \sigma_{\mathbf{ft}} - \mathcal{A}_{\mathbf{xt}} \end{aligned}$$

Then

$$\Delta_{\mathbf{xt}} = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

where

$$\begin{cases} a & = 1 \\ b & = (\sigma_{\mathbf{xt}} - \sigma_{\mathbf{ft}} - \mathcal{A}_{\mathbf{xt}}) + \mathcal{V}_{\mathcal{R}_{\mathbf{xt}}} + \mathcal{V}_{\mathcal{R}_{\mathbf{ft}}} \\ c & = \mathcal{V}_{\mathcal{R}_{\mathbf{xt}}} (\sigma_{\mathbf{xt}} - \sigma_{\mathbf{ft}} - \mathcal{A}_{\mathbf{xt}}) \end{cases}$$